Rekha V.V.I. Questions for 2022 Examination

Answer of below mentioned V.V.I. questions are present in your Rekha Guess Paper Part-III Math-5

1. (a) Let f ∈ R [a, b] and let m, M be the bounds of f on [a, b], than prove:

$$m(b-a) \leq \int_a^b f(x)dx \leq M(b-a) \text{ if } b \geq a \text{ and}$$

$$m(b-a) \geq \int_a^b f(x)\,dx \geq M(b-a) \text{ if } b \leq a.$$ **V. V. I.** **12**

(b) Show that if f is defined on [a, b] by $f(x) = k, \forall x \in. [a, b]$, where k is a constant thus f is R -integrable on [a, b] and

$$\int_a^b k dx = k(b-a)$$ **V. V. I.** **16**

2. (a) Establish the equivalence of bound definition and limit definition of Riemann-integration. **5**

(b) Show that the function f(x) defined in the interval [0, 1] such that:

$$f(x) = \frac{1}{2^n} \text{ where } \frac{1}{2^{n+1}} < x \leq \frac{1}{2^n} \quad f(0) = 0 \text{ where } n = 0, 1, 2, 3$$

........is integrable over [0, 1] and evaluate $\int_0^1 f(x)\,dx$. **13**

3. (a) If f is continuous on [a, b] then prove that it is R-integrable on [a, b]. **V. V. I.** **10**

(b) Give an example of a bounded function which is not R-integrable. **V. V. I.** **8**

4. (a) If f is monotonic on [a, b] then show that f is R-integrable on [a. b]. **12**

(b) State and prove 1st mean value theorem. **17**

5. (a) State Dirichlet's test for convergence of an improper integrals and hence test the convergence of the integral:

$$\int_a^\infty \frac{1}{\sqrt{x}} \sin x \, dx, \ a > 0$$ **V. V. I.** **20**

(b) Test the convergence of the integral

$\int_0^\infty \frac{x^{2m}}{1+x^{2n}}$ dx, where m and n are positive integers. **V. V I.** 21

(c) Discuss the convergence of $\int_0^{\pi/2}$ log sinx dx. 25

6. (a) State and prove Implicit function theorem. **V. V. I.** 37
(b) State and prove Schwartz's theorem. **V. V. I.** 34
(c) Show that $f(x, y) = xy\frac{x^2 - y^2}{x^2 + y^2}$ if $x^2 + y^2 \neq 0$
$= 0$ if $(x, y) = (0, 0)$ is differentiable at the origin . **V. V. I.** 29

7. (a) Obtain Cauchy-Riemann equations in polar form. **V. V. I.** 50
(b) Prove: $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4\frac{\partial^2}{\partial z \partial \bar{z}}$ **V. V. I.** 53
(c) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although Cauchy Riemann equations are satisfied at that point. **V. V. I.** 52

8. (a) Prove that at each point z of a domain where f (z) is analytic and $f'(z) \neq 0$, the mapping $w = f(z)$ is conformal. **V. V. I.** 62
(b) Find the fixed points and normal form of the bilinear transformation $w = \frac{z}{z-2}$ **V. V. I.** 65

9. (a) Prove that every bilinear transformation maps circles or straight lines into circles and straight-lines. **V. V. I.** 60
(b) Find the bilinear transformation that maps the points $z_1 = \infty$, $z_2 = i$, $z_3 = 0$ into the, points $w_1 = 0$, $w_2 = i$, $w_3 = \infty$. **V. V. I.** 61

10. (a) Define continuity and differentiability of a complex function f(z) in a domain D. Prove that differentiability implies continuity. 44
(b) Prove that the function $f(z) = |z|^2$ is continuous everywhere but is nowhere differentiable except at the origin. 46

11. (a) Define metric space with suitable example. Given any three points x, y, z in a metric space (X, d).
Prove: $|d(x, z) - d(y, z)| \leq d(x, y)$. **V. V. I.** 70,74
(b) Prove that the mapping $d : R^2 \times R^2 \to R$ defined by $d(x, y) = |x_1 - y_1| + |x_2 - y_2|$, where $x = (x_1, x_2)$, $y = (y_1, y_2) \in R^2$ is a metric on R^2. **V. V. I.** 73
(c) Let (X, d) be a metric space and $d^*(x, y) = \min\{1, d(x, y)\}$. Prove that d* is a metric for x. **V. V. I.** 76

12. (a) Define Cauchy sequence and prove that every convergent sequence $\{x_n\}$ of points of a metric space (E, d) is a Cauchy sequence, but the converse is not true in general. **V. V. I.** **82**
    (b) Prove that an open sphere in a metric space (B, d) is an open set. **V. V. I.** **78**
13. (a) Define complete metric space. Prove that a subspace Y of a complete metric space (X, d) is complete if and only if Y is closed. **83**
    (b) Prove that every metric space is a Hausdorff space. **74**
    (c) State and prove Baire's Category theorem. **92**
14. Define a topological space with an example. Prove that in a topological space (X; τ), an arbitrary intersection of closed sets is closed and finite union of closed sets is closed. **V. V. I.** **102**

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MATHS - 5 (Hons.) (2021)

9. (a) Let M be a non-empty set. Then a mapping d of $M \times M$ into R is a metric on M iff
(i) $d(x, y) = o \Rightarrow x = y$ for all $x, y \in M$ 75
(ii) $d(x, z) \leq d(x, y) + d(z, y)$ for all $x, y, z \in M$
(b) Let (E, e) be a metric space. Then (E, d) is a metric space where d is defined by $d(x, y) = \dfrac{e(x, y)}{1 + e(x, y)}$ for all x, y of E. 75

10. (a) Let (x, d) be a metric space, then prove that every closed sphere is X is a closed set relative to the d-metric topology for X. 80
(b) In a metric space (x, d) prove that the intersection of two open sets is open. 79

11. (a) State and prove cantor's Intersection theorem. 86
(b) Show that any contraction map T on a metric space (E, d) is uniformly continuous. 86

12. Prove that every compact metric space is complete. 91

MATHS - 5 (Hons.) (2020)

1. (a) Let f $\in$ R [a, b] and let m, M be the bounds of f on [a, b], than prove:

$$m(b-a) \leq \int_a^b f(x)dx \leq M(b-a) \text{ if } b \geq a \text{ and}$$

$$m(b-a) \geq \int_a^b f(x)\,dx \geq M(b-a) \text{ if } b \leq a.$$ 12

(b) Show that if f is defined on [a, b] by $f(x) = k, \forall x \in. [a, b]$, where k is a constant thus f is R -integrable on [a, b] and

$$\int_a^b k dx = k(b-a)$$ 16

2. (a) If f is continuous on [a, b] then prove that it is R-integrable on [a, b]. 10
(b) Give an example of a bounded function which is not R-integrable. 8

3. (a) State Dirichlet's test for convergence of an improper integrals and hence test the convergence of the integral:

$$\int_a^\infty \frac{1}{\sqrt{x}} \sin x\, dx, \ a > 0$$ 20

(b) Test the convergence of the integral

$$\int_0^\infty \frac{x^{2m}}{1+x^{2n}}\,dx,$$ where m and n are positive integers. 21

4. State and prove Implicit function theorem. 37
5. (a) Obtain Cauchy-Riemann equations in polar form. 50

(b) Prove: $\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}=4\frac{\partial^2}{\partial z\partial \bar{z}}$ 53

6. (a) Prove that at each point z of a domain where f (z) is analytic and f ' (z) $\neq$ 0, the mapping w = f (z) is conformal. 62

(b) Find the fixed points and normal form of the bilinear transformation $w = \frac{z}{z-2}$ 65

7. (a) Prove that every bilinear transformation maps circles or straight lines into circles and straight-lines. 60

(b) Find the bilinear transformation that maps the points z_1 = ∞, z_2 = i, z_3 = 0 into the, points w_1 = 0, w_2 = i, w_3= ∞. 61

8. (a) Prove that continuity is necessary but not sufficient condition for the existence of a finite derivative of analytic function.

(b) Show that the function f(z) = z is not differentiable at any point.

9. (a) Define metric space with suitable example. Given any three points x, y, z in a metric space (X, d).

Prove: $|d(x, z) - d(y, z)| \leq d(x, y)$. 70,74

(b) Prove that the mapping $d : R^2 \times R^2 \to R\cdot$ defined by d (x, y) = $|x_1 - y_1| + |x_2 - y_2|$, where x = (x_1, x_2), y = $(y_1, y_2) \in R^2$ is a metric on R^2. 73

10. (a) Define Cauchy sequence and prove that every convergent sequence $\{x_n\}$ of points of a metric space (E, d) is a Cauchy sequence, but the converse is not true in general. 82

(b) Prove that an open sphere in a metric space (B, d) is an open set. 78

11. (a) Define complete metric space. Prove that a subspace Y of a complete metric space (X, d) is complete if and only if Y is closed. 83

(b) State and prove Cantor's intersection theorem. 86

12. Define a topological space with an example. Prove that in a topological space (X; τ), an arbitrary intersection of closed sets is closed and finite union of closed sets is closed. 102

MATHS - 5 (Hons.) (2019)

Answer any six questions.

1. (a) Eatablish the equivalence of bound definition and limit definition of Riemann-integration. 5

(b) Show that the function f(x) defined in the interval [0, 1] such that:

$$f(x) = \frac{1}{2^n} \text{ where } \frac{1}{2^{n+1}} < x \leq \frac{1}{2^n} \quad f(0) = 0 \text{ where } n = 0, 1, 2, 3$$

........is integrable over [0, 1] and evaluate $\int_0^1 f(x)\,dx$. 13

2. (a) If f is monotonic on [a, b] then show that f is R-integrable on [a. b]. 12

(b) State and prove 1st mean value theorem. 17

3. (a) State and prove Abel's test for the convergence of the integral of a product of two functions. 19

(b) Discuss the convergence of $\int_0^{\pi/2} \log \sin x\, dx$. 25

4. (a) State and prove Schwartz's theorem. 34

(b) Show that $f(x, y) = xy\frac{x^2 - y^2}{x^2 + y^2}$ if $x^2 + y^2 \neq 0$

= 0 if (x, y) = (0, 0) is differentiable at the origin 29

5. (a) Define analytic function. Find the necessary and sufficient condition for f(z) to be analytic. 47

(b) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although Cauchy Riemann equations are satisfied at that point. 52

6. (a) Define bilinear transformation. Prove that the resultant of two bilinear transformation is a linear transformation. 56

(b) Prove that the cross-ratio of four points is invariant under a bilinear transformation. 64

7. (a) Define continuity and differentiability of a complex function f(z) in a domain D. Prove that differentiability implies continuity. 44

(b) Prove that the function $f(z) = |z|^2$ is continuous everywhere but is nowhere differentiable except at the origin. 46

8. (a) Define inverse points. Show that the inverse of a point *a* with respect to the circle |z - c| = r is the point $c + \frac{r^2}{\overline{a} - \overline{c}}$. 68

(b) Obtain the condition for four points to be concyclic. 66

9. (a) Let X be of a non-empty set and d be a real valued function of X × X into R. Then prove that d is a metric iff :

(i) d (x, y) = 0 ⇔ x = y

(ii) d (x, y) ≤ d (x, z) + d (y, z) 75

(b) Let (X, d) be a metric space and d* (x, y) = min {1, d(x, y)}. Prove that d* is a metric for x. 76

10. (a) Let (X, d) be a metric space, then prove that every closed sphere in X is a closed set relative to the d-metric topology for X. **80**

    (b) In a metric space (X, d) prove that the intersection of two open sets is open. **79**

11. (a) Prove that every metric space is a Hausdorff space. **74**

    (b) State and prove Baire's Category theorem. **92**

12. Prove that every compact metric space is complete. **91**

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Rekha V.V.I. Questions for 2022 Examination

Answer of below mentioned V.V.I. questions are present in your Rekha Guess Paper Part-III Math-6

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MATHS - 6 (Hons.) (2021)

1. Define inner automorphism. Prove that the set I (G) of all inner automorphism of a group G is a normal sub group of the group of its automorphism and isomorphic to the quotient group G/Z of G, where Z is the centre of G. 7,8
2. (a) Define normalizer of an element of a group. Prove that the normalizer N (a) of a $\in$ G is a sub group of G. 14
   (b) State and prove class equation of a finite group. 15
3. (a) State and prove Cauchy's theorem for finite abelian group. 19
   (b) If H is a p-sylow sub group of G and $x \in$ G then prove that $x^{-1} Hx$ is also a p-sylow sub group of G. 23
4. (a) Suppose R is a ring, S and ideal of R. Let f be a mapping from R to R/S defined by $f(a) = S + a, \forall a \in R$. Then prove that f is a homomorphism of R onto R/S. 28
   (b) Prove that an ideal S of a commutative ring R with unity is maximal if and only if the residue class ring R/S is a field. 28
5. (a) Define Euclidean Ring. Prove that the ring of polynomials or a field is n Euclidean ring. 33
   (b) Prove that every Euclidean ring is a principal ideal ring. 35
6. Define Unique Factorization Domain. Prove that every Euclidean Domain is a unique Factorization Domain. 46
7. Define vector space.Let V(F) be a vector space and 0 be the zero vector of V then prove :
   (a) $a.0 = 0, \forall a \in F$
   (b) $0.\alpha = 0, \forall \alpha \in V$
   and (c) $a.\alpha = 0 \Rightarrow a = 0$ or $\alpha = 0$ 50
8. (a) If W_1 and W_2 are sub spaces of a vector space V(F) then prove :
   (i) $W_1 + W_2$ is a sub space of V(F) 53
   and (ii) $L(W_1 \cup W_2) = W_1 + W_2$
   (b) Show that the vectors (1,2,1), (2,1,0), (1,-1,2) form a basis of R^3. 58
9. Prove that the set S of all linear transformations from a vector space V(K) into a vector space U(K) is a vector space over the field F relative to the operations of vector addition and scalar

multiplication defined as :

$(T_1 + T_2)(x) = T_1(x) + T_2(x)$

and $(a T_1)(x) = a T_1(x), \forall x \in V, a \in k$

and $T_1, T_2 \in S$

10. (a) If A is a non-singular matrix. Then show that the eigen values of A^{-1} are the reciprocals of the eigen values of A and conversely. 74

(b) Find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$$

11. (a) Introduce the concept of an inner-product space and prove that every inner product space is a normed linear space but not conversely. 78

(b) Construct a Banach space which is not a Hilbert space. 85

12. (a) Introduce the concepts of module and submodule with examples illustrating them. 89

(b) Prove that every abelian group G is a module over the ring of integers. 90

MATHS - 6 (Hons.) (2020)

1. (a) Prove that the set of all automorphisms of a group forms a group with respect to the composite composition. 5

(b) Prove that for an abelian group, the only inner automorphism is the identity mapping whereas for non abelian groups there exist non trivial automorphism.

2. (a) Prove that every group of prime order is cyclic. 20

(b) Prove that if G is a group of order p^n then its centre $z \neq \{e\}$, where p is a prime order. 16

3. State and prove Sylow's theorem on groups. 21

4. (a) State and prove fundamental theorem of homomorphism of ring. 24

(b) Prove that the Kernel of a homomorphism of a ring R into a ring S is an ideal in R. 25

5. Prove that every integral domain can be embedded into a field. 30

6. State and prove Einstein criterion for irreducibility of a polynomial. 43

7. (a) Prove that necessary and sufficient condition for a non empty subset w of a vector space v (F) to be a subspaces of v is $a, b \in F, \alpha, \beta \in w \Rightarrow a\alpha + b\beta \in w$. 52

(b) Prove that intersection of any two subspaces of a vector space is a subspace. 53

8. (a) If w_1 and w_2 are subspaces of a finite dimensional vector space v (F), then prove that $\dim(w_1 + w_2) = \dim(w_1) + \dim(w_2) - \dim(w_1 \cap w_2)$. 57

(b) Show that the set S = {(1, 2, 1), (3, 1,5), (3, - 4, 7)} is linearly dependent when $S \leq V_3(R)$.

9. Let U and V be vector spaces over the field F and let T be a linear transformation from U into V suppose that U is finite. Then prove that Rank (T) + nullity (T) = dim U. 60

10. (a) Prove that the characteristics roots of a real symmetric matrix are all real. 75

(b) Find the eigen values of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ 77

11. (a) State and prove Schwarz's inequality for an inner product space. 79

(b) If α, β are vectors in an inner product space V then prove that: $\|\alpha + \beta\| \leq \|\alpha\| + \|\beta\|$ 80

12. (a) If A and B are two submodules of an R-module M with $B \subseteq A$. Then prove that M/A is isomorphic to some quotient module of M/B. 96

(b) Prove that Kernel of homomorphism of module is submodule. 94

MATHS - 6 (Hons.) (2019)

Answer any six questions.

1. (a) Define an automorphism on a group G. Prove that a mapping f defined on a group G by $f(x) = x^{-1}$, for all $x \in G$ is an automorphism on G if and only if G is abelian. 5

(b) Introduce the concept of an inner automorphism of a group. Prove that the set of all inner automorphisms of a group G is a normal subgroup of the group of all automorphisms of G. 7

2. (a) Define the centre Z of a group G and prove that if G/Z is cyclic then G is abelian. 15

(b) Let p be a prime number then G be a group of order p^2. Prove that G is abelian. 17

3. (a) State and prove Cauchy's theorem for finite abelian groups. 19

(b) If H is a p–Sylow subgroup of a group G and $x \in G$, then prove that $x^{-1} H x$ is also a p–Sylow subgroup of G. 23

4. (a) Introduce the concept of an ideal in a ring. If R is a commutative ring with unity element 1 and $a_0 \in R$ then prove that $a_0 R = \{a_0 . r \mid r \in R\}$ is an ideal in R.

(b) State and prove division algorithm for a polynomial ring F[x] over a field F.

5. Define the quotient ring R/I of a given ring R with respect to a given ideal I of R. Prove that if R is commutative then so is R/I and if R has a unity element 1 and I is a proper ideal then R/I has a unity element. 26

6. Define a Unique Factorisation Domain and prove that every Euclidean Domain is a Unique Factorisation Domain. 46

7. (a) Show that the set D[0, 1] of all real valued differentiable functions on [0, 1] is a real vector space under pointwise linear operations on D[0,1]. 50

(b) Let V be a vector space of dimension n. Prove that any set of n linearly independent elements of V is a basis of V. 56

8. (a) If a vector space V over a field F has dimension n with $n > 0$, then prove that V is isomorphic to the vector space $V_n(F)$ of all n–tuples of scalars. 72

(b) Prove that the vectors (x_1, x_2) and (y_1, y_2) in $V_2(F)$ are linearly dependent if and only if $x_1y_2 - x_2y_1 = 0$.

9. Let V and V' be vector spaces over a field F. If dim V = n and $T : V \rightarrow V'$ is a linear transformation of rank r then prove that T has nullity (n - r).

10. (a) Define Eigen Values and Eigen vectors of a linear operator T on a finite dimensional vector space. Prove that the eigen vectors of T belonging to different eigen values of T are linearly independent. 73

(b) Find the eigen values of the matrix

$$A = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$$ 77

11. (a) State and prove Cauchy-Schwarz inequality in a Hilbert space. 85

(b) Construct a Banach space which is not a Hilbert space.

12. Introduce the concept of sub-module of a given module. If A and B are sub-modules of a module M then prove that :

(a) $A \cap B$ is a sub-module of M.

(b) $A + B = \{a + b \mid a \in A, b \in B\}$ is a sub-module of M. 89,90

(c) $(A + B)/B$ is isomorphic to $B/(A \cap B)$.

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Rekha V.V.I. Questions for 2022 Examination

Answer of below mentioned V.V.I. questions are present in your Rekha Guess Paper Part-III Math-7

1. (a) Prove that any system of forces, acting in one plane upon a rigid body can be reduced to either a single force or a single couple. **V. V. I.** 5

   (b) Two system of forces P, Q, R and P', Q', R' act along the sides BC, CA, AB of a triangle ABC. Prove that their resultants will be parallel if (QR'-Q'R) sin A + (RP'-R'P) sin B + (PQ'-P'Q) sin C = 0. **V. V. I.** 9

2. (a) Find the equation of the line of action of the resultant of a system of coplanar forces acting on a rigid body. 6

   (b) Forces P, Q, R act along the sides of the triangle formed by the lines x = 0, y = 0 and $x\cos\theta + y\sin\theta = p$, axes being rectangular. Find the magnitude of the resultant and equation of its line of action. 8

3. (a) Find the forces which may be omitted in forming the equation of virtual work. **V. V. I.** 14

   (b) Two equal uniform rods AB and AC, each of length 2b, are jointed at A and rest on a smooth vertical circle of radius a. Show that if 2θ be the angle between them then $b\sin^3\theta = a\cos\theta$. **V. V. I.** 19

   (c) The middle points of the opposite sides of a jointed quadrilateral are connected by light rods of lengths l and l'. If T and T' be the tensions of these rods, then prove that $\frac{T}{l} + \frac{T'}{l'} = 0$. **V. V. I.** 16

4. (a) Derive the equation of common catenary in the form $y = c\cosh x/c$. 34

   (b) Prove the following for a common catenary :

   (i) $y = c\sec\psi$ (ii) $y^2 = s^2 + c^2$ 34

   (c) Show that the length of a heavy endless chain which will hang over circular pulley of radius a so as to be in contact with two third of the circumference is : $a\left[\frac{3}{\log(2+\sqrt{3})} + \frac{4\pi}{3}\right]$ 41

5. Establish the energy test for stability. A solid frustum of a paraboloid of revolution of height h and latus rectum 4a rests with its vertex on the vertex of a paraboloid of revolution, whose latus rectum is 4b show that the equilibrium is stable if $h < \frac{3ab}{a+b}$. **V.V.I.** 21

6. (a) Find the amplitude and frequency of the combined motion of two simple harmonic motions of the same period and in the same straight line. 47

(b) A particle whose mass is m, is acted upon by a force $m\mu(x+\frac{a^4}{x^3})$ towards the origin O; if it starts from rest at a distance a, show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$. 54

7. (a) State Kepler's law of planetary motion and deduce the third law from Newton's law of gravitation. **V. V. I.** 68

(b) If V_1 and V_2 be the linear velocity of a planet when it is respectivley nearest and farthest from the sun, then prove that $(1-e)V_1=(1+e)V_2$. **V. V. I.** 76

(c) Prove that the extension of a heavy elastic string of weight w and natural length l hanging from one end and supporting a weight w' at the $\frac{l}{\lambda}$ (w' $+\frac{1}{2}$ w), where λ is the modulus of elasticity of the string. **V. V. I.** 61

8. (a) Find differential of central orbit in polar co-ordinates. **VV. I.** 65

(b) A particle describes the curve $v^n = a^n. \cos n\theta$ under a force P to the pole. Find the law of force. **V. V. I.** 70

9. (a) Find the work done in extending a light elastic string to double its length. **V. V. I.** 63

(b) Define minimum time of oscillator of a compound pendulum. **VVI** 82

(c) Determine the motion of a rigid body acted on by the force of gravity only and moving about a fixed horizontal axis. **VVI** 81

10. (a) Define and interpret geometrically the scalar product of three vectors. **V. V. I.** 84

(b) Prove : $\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})=0$ **V. V. I.** 87

(c) Prove : $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}]=2[\vec{a}\ \vec{b}\ \vec{c}]$ **V. V. I.** 87

11. (a) Show that the necessary and sufficient condition for the vector function $\vec{v}$ of scalar variable t to have constant magnitude is $\vec{v}.\frac{d\vec{v}}{dt}=0$ **V. V. I.** 91

(b) If $\vec{a}$ is a unit vector, prove $\left|\vec{a}\times\frac{d\vec{a}}{dt}\right|=\left|\frac{d\vec{a}}{dt}\right|$ **V. V. I.** 92

12. (a) If $\vec{v} \times \frac{d\vec{v}}{dt} = 0$, then show that $\vec{v}$ (t) is a constant vector. **92**

(b) Evaluate : $\frac{d}{dt}\left(r.\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}\right)$ **94**

13. (a) Prove : Curl $(\phi \vec{a}) = \phi \operatorname{curl} \vec{a} + (\operatorname{grad}. \phi) \times \vec{a}$ **V. V. I.** **101**

(b) Prove : Div. $(\operatorname{Curl} \vec{v}) = 0$ **V. V. I.** **99**

14. (a) Prove : div. $(\vec{a} \pm \vec{b}) = div.\vec{a} \pm div.\vec{b}$ **V. V. I.** **95**

(b) Prove : Curl (grad . ϕ) = 0 **V. V. I.** **101**

15. State and prove Green's theorem. **V. V. I.** **108**

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MATHS - 7 (Hons.) (2021)

Answer any SIX questions.

1. (a) Obtain the general conditions of equilibrium of a system of forces acting in one plane upon a rigid body. 10

   (b) Three forces P,Q,R act along the sides of the triangle formed by the lines $x+y=1, y-x=1$ and $y=z$. Find the equation to the line of action of the resultant. 7

2. (a) State and prove the principle of virtual work for any system of forces in one plane. 12

   (b) A regular hexagon ABCDEF consist of six equal uniform rods, each of the weight w, freely joined together. The hexagon rest in a vertical plane and AB is in contact with a horizontal table. If C and F be connected by a light string. Prove that the tension is $w\sqrt{3}$. 37

3. (a) For a common catenary prove that $x = c \log(\sec\psi + \tan\psi)$. 36

   (b) A telegraph wire stretched between two poles at a distance a metre apart, sangs n metre in the middle. Prove that the tension at the end in approximately $w\left(\frac{a^2}{8n}+\frac{7}{6}n\right)$ where w is the weight per unit length. 42

4. Find the condition of stability for a body with one degree of freedom. 20

5. (a) Find the time period, amplitude and frequency in as S.H.M. 45

   (b) A particle starts with a given velocity V and moves under a retardation equal to K times the space described. Show that the distance traversed before it comes to rest is $\frac{V}{\sqrt{K}}$ 56

6. (a) Prove that the work done against the tension in stretching a light elastic string in equal to the product of its extension and the mean of the initial and final tensions. 58

   (b) A mass hangs from a fixed point by a straight string and is given a small vertical displacement. Show that the motion in S.H.M. 63

7. Prove that the rate of change of momentum of a body in any given direction is equal to the resolve part of the external forces in the same direction. 77

8. (a) State and prove Kepler's law of central orbit. 68

   (b) A particle describes the centre $P^2 = ar$ under the force P to the pole. Find the law of forces. 71

9. (a) Prove that $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\cdot\vec{b}-(\vec{a}\cdot\vec{b})\cdot\vec{c}$ 86

(b) Show that $[\vec{a}\times\vec{b} \quad \vec{b}\times\vec{c} \quad \vec{c}\times\vec{a}]=[\vec{a}\ \vec{b}\ \vec{c}]^2$ 87

10. (a) If $\vec{a}$ and $\vec{b}$ are differentiable vector functions of a scalar t, then prove that

$$\frac{d}{dt}(\vec{a}\times\vec{b}) = \vec{a}\times\frac{d\vec{b}}{dt}+\frac{d\vec{a}}{dt}\times\vec{b}$$ 93

(b) Find the value of $\frac{d}{dt}[\vec{a},\vec{b},\vec{c}]$ 93

11. (a) Prove that $div\,(\vec{a}\times\vec{b})=\vec{b}\cdot(Curl\,\vec{a})-\vec{a}\cdot(Curl\,\vec{b})$ 100

(b) Prove that $(\vec{r}\cdot\nabla)\cdot\varphi=\vec{r}(\nabla\varphi)$ 98

12. State and prove Stoke's theorem. 110

MATHS - 7 (Hons.) (2020)

1. (a) Prove that any system of forces, acting in one plane upon a rigid body can be reduced to either a single force or a single couple. 5

(b) Two system of forces P, Q, R and P', Q', R' act along the sides BC, CA, AB of a triangle ABC. Prove that their resultants will be parallel if (QR'-Q'R) sin A + (RP'-R'P) sin B + (PQ'-P'Q) sin C = 0. 9

2. (a) Find the forces which may be omitted in forming the equation of virtual work. 14

(b) Two equal uniform rods AB and AC, each of length 2b, are jointed at A and rest on a smooth vertical circle of radius a. Show that if 2θ be the angle between them then $b\sin^3\theta = a\cos\theta$. 19

3. (a) Derive the equation of common catenary in the form y = c cos h x/c 34

(b) If α and β be the angles which a string of length l makes with the vertical at the points of support, show that the height of one point above the other is $\dfrac{l\cos\frac{1}{2}(\alpha+\beta)}{\cos\frac{1}{2}(\beta-\alpha)}$

4. Establish the energy test for stability. A solid frustum of a paraboloid of revolution of height h and latus rectum 4a rests with its vertex on the vertex of a paraboloid of revolution, whose latus rectum is 4b show that the equilibrium is stable if $h<\dfrac{3ab}{a+b}$. 21

5. (a) Find the amplitude and frequency of the combined motion of two simple harmonic motions of the same period and in the same straight line. 47

(b) A particle whose mass is m, is acted upon by a force $m\mu\left(x+\frac{a^4}{x^3}\right)$ towards the origin O; if it starts from rest at a distance a, show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$. 54

6. State Kepler's law of planetary motion and deduce the third law from Newton's law of gravitation. 68

7. (a) Find the differential of the central orbit in polar co-ordinates. 65

(b) A particle describes the curve $v^n = a^n . \cos n\theta$ under a force P to the pole. Find the law of force. 70

8. (a) Show that if no external forces act on the system of particle moving on a straight line the, centre of inertia is either at rest or moves with uniform velocity.

(b) Find the work done in extending a light elastic string to double its length. 63

9. (a) Define and interpret geometrically the scalar product of three vectors. 84

(b) Prove : $\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})=0$ 87

10. (a) Show that the necessary and sufficient condition for the vector function $\vec{v}$ of scalar variable t to have constant magnitude is $\vec{v}.\frac{d\vec{v}}{dt}=0$ 91

(b) If $\vec{a}$ is a unit vector, prove $\left|\vec{a}\times\frac{d\vec{a}}{dt}\right|=\left|\frac{d\vec{a}}{dt}\right|$ 92

11. (a) Prove : Curl $(\phi\vec{a})=\phi$ curl $\vec{a}$ + (grad. ϕ) $\times\vec{a}$ 101

(b) Prove : Div. $(\text{Curl}\ \vec{v})=0$ 99

12. State and prove Green's theorem. 108

MATHS - 7 (Hons.) (2019)

Answer any six questions.

1. (a) Find the equation of the line of action of the resultant of a system of coplanar forces acting on a rigid body. 6

(b) Forces P, Q, R act along the sides of the triangle formed by the lines x = 0, y = 0 and $x\cos\theta + y\sin\theta = p$, axes being rectangular. Find the magnitude of the resultant and equation of its line of action. 8

2. (a) State and prove the principles of virtual work for a system of coplanar forces. 12

(b) The middle points of the opposite sides of a jointed quadrilateral are connected by light rods of lengths l and l'. If T and T' be the tensions of these rods, then prove that $\frac{T}{l}+\frac{T'}{l'}=0$. 16

3. (a) Prove the following for a common catenary :
(i) $y = c \sec \psi$ (ii) $y^2 = s^2 + c^2$ 34
(b) Show that the length of a heavy endless chain which will hang over circular pulley of radius a so as to be in contact with two third of the circumference is : $a\left[\frac{3}{\log(2+\sqrt{3})}+\frac{4\pi}{3}\right]$ 41

4. Find the conditions of stability for a body with one degree of freedom. 20

5. (a) Define a simple harmonic motion. Prove that the time period of a S. H. M. is independent of its amplitude. 45
(b) A body moves from rest from a point O so that its acceleration after t seconds from O is $\frac{1}{(t+2)^2}$. Find the distance described in 9 seconds and its velocity then.

6. (a) If V_1 and V_2 be the linear velocity of a planet when it is respectivley nearest and farthest from the sun, then prove that $(1 - e) V_1 = (1 + e)V_2$. 76
(b) Prove that the extension of a heavy elastic string of weight w and natural length l hanging from one end and supporting a weight w' at the $\frac{l}{\lambda}$ (w' + $\frac{1}{2}$ w), where λ is the modulus of elasticity of the string. 61

7. State D' Alembert's principle and prove that the rate of change of momentum of a body in any given direction is equal to the resolved part of the external forces in the same direction. 77

8. (a) Define minimum time of oscillator of a compound pendulum. 82
(b) Determine the motion of a rigid body acted on by the force of gravity only and moving about a fixed horizontal axis. 81

9. (a) Prove : $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$ 87
(b) Prove : $b^2\vec{a} = (\vec{a}.\vec{b})\vec{b}+\vec{b}\times(\vec{a}\times\vec{b})$

10. (a) If $\vec{v}\times\frac{d\vec{v}}{dt}=0$, then show that $\vec{v}$ (t) is a constant vector. 92
(b) Evaluate : $\frac{d}{dt}\left(r.\frac{d\vec{r}}{dt}\times\frac{d^2\vec{r}}{dt^2}\right)$ 94

11. (a) Define divergence and curl of a vector field.
Prove : div. $(\vec{a}\pm\vec{b}) = div.\vec{a}\pm div.\vec{b}$ 95
(b) Prove : Curl (grad . ϕ) = 0 101

12. State and prove Stoke's theorem. 110

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MATHS - 8 (Hons.) (2021)

Spherical Trigonometry & Astronomy

1. (a) Find the value of cosine of an angle of a spherical triangle in terms of cosines and sines of the sides. 51

   (b) Prove that in spherical triangle

$$\tan\frac{A}{2}=\sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s\sin(s-a)}}$$ 52

2. (a) In spherical triangle prove that

$$\tan\frac{A+B}{2}=\frac{\cos\frac{a-b}{2}}{\cos\frac{a+b}{2}}\cot c/2$$ 54

   (b) In any spherical triangle prove that :

$$\frac{\sin(A+B)}{\sin C}=\frac{\cos a+\cos b}{1+\cos c}$$ 57

3. (a) In a spherical triangle ABC, in which $\angle c=\frac{\pi}{2}$ prove that

$$\tan\frac{A}{2}\sin a=\sin c-\cos a\cos b$$ 59

   (b) In a spherical triangle ABC, if $A=\frac{\pi}{2},B=\frac{\pi}{3},C=\frac{\pi}{2}$ show that

$$a+b+c=\frac{\pi}{2}$$ 59

4. (a) Explain Rising and Setting of Stars. 71

   (b) If h be the hour angle of a star at rising, then prove that

$$\tan^2\frac{h}{2}=\frac{\cos(Q-\delta)}{\cos(Q+\delta)}$$ 73

5. (a) What is the effect of refraction on sunrise and sunset ? 100

   (b) If r is the horizontal refraction, show that on account of this the point of the compass where the sunrises is shifted by

$$\frac{\sin Q}{\cos(Q-\delta)\cos(Q+\delta)}\cdot r,$$ where Q is latitude. 101

6. (a) Obtain Kepler's Equation E = me sin E where m is the mean anomaly and E is the eccentric anomaly. 91

(b) Prove that if the forth and higher powers of e are neglected, then prove that $E = m + \frac{e\sin m}{1-e\cos m} - \frac{1}{2}\left\{\frac{e\sin m}{1-e\cos m}\right\}^m$ is a solution of Kepler's equation. 92

7. (a) Prove that the equation of time vanishes four times in an year. 83

(b) Prove that the equation of time due to obliquity of ecliptic is max. when the longitude Θ of the sun is given by sin $\Theta = \frac{1}{\sqrt{2}}\sec\frac{\in}{2}$ 87

8. Discuss the effect of aberration on latitude and longitude of a star. 105

9. Find the nutation in right ascension and declination. 111

10. Find Geocentric parallax in right ascension and declination. Earth taken as spheroid. 107

MATHS - 8 (Hons.) (2020)

Answer any five questions.

1. (a) State Hausdorff 's axiom system for topological space. Illustrate with a suitable example.

(b) Prove that every metric space (X, d) is a metrizable space with the topology induced by the metric d.

2. Define convergence of a sequence in a topological space. Prove that in a Hausdorff space every convergent sequence has a unique limit. Also show that the condition for a topological space to be a Hausdorff space is not necessary for the uniqueness of limit of convergent sequences.

3. (a) Let (X, τ_1) and (Y, τ_2) be two topological spaces. Then prove that a function f: X→ Y is τ_1,τ_2 continuous if and only if the inverse image under f of every -closed sub-set of Y is a τ_1- closed subset of X.

(b) Prove that a necessary and sufficient condition for a one to one onto mapping f: X → Y, where X and Y are topological spaces to be homeomorphism is that $f(\bar{A}) = \overline{f(A)}$ for every $A \subseteq X$.

4. Define compact topological space. Prove that every closed subset of a compact space is compact but every compact subset of a topological space is not necessarily closed.

5. (a) Let (X, τ_1) and (Y, τ_2) be two topological spaces and let f be a continuous mapping of X into Y, then prove that for every compact subset E of X; f (E) is a compact subset of Y.
   (b) Show that (R, U) is not a compact space, where U is the usual topology on R.
6. (a) Let X be a topological space. If $\{A_i\}$ is a non-empty family of connected subsets of X such that $\bigcap_i A_i \neq \phi$, then prove A $= \bigcap$ is also a connected subset of X.
   (b) Construct a topological space which is compact but not connected.
7. (a) Prove that a continuous image of a connected set is also connected.
   (b) Prove that a subspace X of the real line R with usual topology is connected if and only if X is an interval.
8. (a) Define T_1-space. Prove that the property of being a T_1-space is both topological and hereditary.
   (b) Prove that a topological space (X, τ) is a T_1- space if and only if every singleton sub-set {x} of of X is a closed set.
9. (a) Define T_2-space. Prove that every subspace of T_2-space is T_2-space.
   (b) Prove that every convergent sequence in a Hausdorff space has a unique limit.
10. Show that the following statements for a metrizable space X are equivalent:
    (a) X is a second countable space
    (b) X is a Lindelof space
    (c) X is a separable space

MATHS - 8 (Hons.) (2019)

Answer any six questions.

1. (a) Find the value of cosine of the side of a spherical triangle in terms of cosines and sines of angle. **51**
   (b) In a spherical triangle prove the formula :

$$\sin\frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b.\sin c}}$$ **52**

2. (a) In a spherical triangle, prove that: $\tan\frac{A+B}{2} = \frac{\cos\frac{a-b}{2}}{\cos\frac{a+b}{2}}\cot\frac{C}{2}$ **54**

   (b) In a spherical triangle prove that : $= \frac{\sin\frac{A-b}{2}}{\cos\frac{C}{2}} = \frac{\sin\frac{a-b}{2}}{\sin\frac{C}{2}}$

3. (a) In a spherical triangle, in which $\underline{|C} = \pi/2$ prove that:

$$\tan^2 \frac{a}{2} = \tan \frac{c+b}{2} \tan \frac{c-b}{2}$$ 57

(b) $$\frac{\sin(a-b)}{\sin(a+b)} = \tan \frac{A+B}{2} \tan \frac{A-B}{2}$$ 58

4. (a) What do you mean by twilight ? Find the condition for twilight to last all night. 74

(b) If twilight begins or ends when the sun is 18° below the horizon, show that so long as the sun's declination is less than 18°, all places have a day of more than 12 hours including the twilight. 76

5. (a) Establish Simpsons Hypothesis. 95

(b) What is effect of Refraction on sunrise and sunset ? 100

6. (a) Establish Bessel's formula for correction to the observed time of transit of a star on account of all the three errors a, b and c considered together. 80

(b) Prove that the error in the time of transit of a star due to the three instrumental errors is a minimum for a star whose declination is $\sin^{-1}\left(\frac{a\cos\phi - b\sin\phi}{c}\right)$, where ϕ is the latitude of the observatory and a, b, c are respectively the azimuth, level and collimation error. 82

7. (a) State Kepler's laws of planetary motion and show that Kepler's laws can be deduced from Newton's Laws of gravitation. 88

(b) Prove that: $\tan \frac{v}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2}$ 90

8. (a) Find the equation of time and show that the equation of time vanishes four times in an year. 83

(b) Prove that if the eccentricity of the earth's orbit were zero, the equation of time in minutes would be.

$$\frac{720}{\pi} \tan^{-1}\left[\frac{(1-\cos\in)\tan\theta}{(1+\cos\in)\tan^2\theta}\right]$$ 87

9. Find the effect of aberration on latitude and longitude of a star. 105

10. Find the Geocentric parallax in right ascension and declination when earth is taken as spheroid. 107

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